Int J Theor Phys (2010) 49: 1457–1464
DOI 10.1007/s10773-010-0326-1

Squark Pair Production at Muon Colliders in the MSSM with CP Violation

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Published online: 31 March 2010

Abstract We study the pair production of scalar quark in a muon collider within the MSSM with CP violation. We show that including the CP phases can strongly affect the cross section of the process: $\mu^+\mu^- \to \tilde{q}_i\bar{\tilde{q}}_j$. This could have an important impact on the search for squarks and the determination of the MSSM parameters at future colliders.

Keywords MSSM · CP violation · Muon collider

1 Introduction

The minimal supersymmetric standard model (MSSM) is one of the most promising extensions of the Standard Model. The MSSM predicts the existence of scalar partners to all known quarks and leptons. Each fermion has two spin zero partners called sfermions $\widetilde{f}_L$ and $\widetilde{f}_R$, one for each chirality eigenstate: the mixing between $\widetilde{f}_L$ and $\widetilde{f}_R$ is proportional to the corresponding fermion mass, and so negligible except for the third generation.

Only three terms in the supersymmetric Lagrangian can give rise to CP violating phases, which cannot be rotated away: The superpotential contains a complex coefficient μ in the term bilinear in the Higgs superfields. The soft supersymmetry breaking operators introduce two further complex terms, the gaugino masses $\widetilde{M}_i$, and the left- and right-handed squark mixing term A_q. In the MSSM one has two types of scalar quarks (squarks), $\widetilde{q}_L$ and $\widetilde{q}_R$, corresponding to the left and right helicity states of a quark. The mass matrix in the basis $(\widetilde{q}_L, \widetilde{q}_R)$ is given by [1],

$$M_{\widetilde{q}}^2 = \begin{pmatrix} m_{\widetilde{qL}}^2 & a_q m_q \\ a_q m_q & m_{\widetilde{qR}}^2 \end{pmatrix} = (R^{\widetilde{q}})^+ \begin{pmatrix} m_{\widetilde{q}1}^2 & 0 \\ 0 & m_{\widetilde{q}2}^2 \end{pmatrix} (R^{\widetilde{q}}) \tag{1}$$

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with

$$m^2_{\widetilde{q}_L} = M^2_{\widetilde{Q}} + m^2_Z \cos 2\beta (I^q_{3L} - e_q \sin^2\theta_w) + m^2_q, \tag{2}$$

$$m^2_{\widetilde{q}_R} = M^2_{\{\widetilde{u},\widetilde{D}\}} + e_q m^2_Z \cos 2\beta \sin^2\theta_w + m^2_q, \tag{3}$$

$$a_q = A_q - \mu\{\cot\beta, \tan\beta\}, \tag{4}$$

for {up, down} type squarks, respectively. e_q and I^q_{3L} are the electric charge and the third component of the weak isospin of the squark $\widetilde{q}$, and m_q is the mass of the partner quark. $M_{\widetilde{Q}}$, $M_{\widetilde{u}}$ and $M_{\widetilde{D}}$ are soft SUSY breaking masses, and A_q are trilinear couplings. According to (1) $M^2_{\widetilde{q}}$ is diagonalized by a unitary matrix $R^{\widetilde{q}}$. The weak eigenstates $\widetilde{q}_L$ and $\widetilde{q}_R$ are thus related to their mass eigenstates $\widetilde{q}_1$ and $\widetilde{q}_2$ by

$$\begin{pmatrix} \widetilde{q}_1 \\ \widetilde{q}_2 \end{pmatrix} = R^{\widetilde{q}} \begin{pmatrix} \widetilde{q}_L \\ \widetilde{q}_R \end{pmatrix} \tag{5}$$

with complex parameters, we have

$$R^{\widetilde{q}} = \begin{pmatrix} e^{\frac{i}{2}\phi_{\widetilde{q}}} \cos\theta_{\widetilde{q}} & e^{-\frac{i}{2}\phi_{\widetilde{q}}} \sin\theta_{\widetilde{q}} \\ -e^{\frac{i}{2}\phi_{\widetilde{q}}} \sin\theta_{\widetilde{q}} & e^{-\frac{i}{2}\phi_{\widetilde{q}}} \cos\theta_{\widetilde{q}} \end{pmatrix}, \tag{6}$$

with $\theta_{\widetilde{q}}$ is the squark mixing angle and $\phi_{\widetilde{q}} = \arg(A_q)$. The mass eigenvalues are given by

$$m^2_{\widetilde{q}_{1,2}} = \frac{1}{2}\left(m^2_{\widetilde{q}_L} + m^2_{\widetilde{q}_R} \mp \sqrt{(m^2_{\widetilde{q}_L} - m^2_{\widetilde{q}_R})^2 + 4a^2_q m^2_q}\right). \tag{7}$$

By convention, we choose $\widetilde{q}_1$ to be the lighter mass eigenstate. For the mixing angle $\theta_{\widetilde{q}}$ we require $0 \preceq \theta_{\widetilde{q}} \preceq \pi$. We thus have

$$\cos\theta_{\widetilde{q}} = \frac{-|a_q| m_q}{\sqrt{(m^2_{\widetilde{q}_L} - m^2_{\widetilde{q}_1})^2 + a^2_q m^2_q}}, \qquad \sin\theta_{\widetilde{q}} = \frac{m^2_{\widetilde{q}_L} - m^2_{\widetilde{q}_1}}{\sqrt{(m^2_{\widetilde{q}_L} - m^2_{\widetilde{q}_1})^2 + a^2_q m^2_q}}. \tag{8}$$

In particular, this model shows that the possibility to discover one of the scalar partners of the top quark ($\widetilde{t}_1$) is higher than that of other scalar quarks and the top quark [1]. As well known, CP violation arises naturally in the third generation Standard Model and can appear only through the phase in the CKM-matrix. In the MSSM with complex parameters, the additional complex couplings may lead to CP violation within one generation at one-loop level [2, 3].

A muon collider can be circular and much smaller than e^+e^- or hadron colliders of comparable effective energies. With its expected excellent energy and mass resolution a muon collider offers extremely precise measurements. Moreover, it allows for resonant Higgs production; in particular it may be possible to study the properties of relatively heavy H^0 and A^0 which can hardly be done at any other collider. Information about ongoing work can be found in [4–6].

In this paper we study the squark pair production in $\mu^+\mu^-$ collider within the MSSM with complex parameters. The analytical formulae are derived and numerical results are discussed.

2 Analytical Results

Our terminology and notation are as in [7, 8]. Squark pair production in $\mu^+\mu^-$-annihilation proceeds via the exchange of a photon, a Z boson, or a neutral Higgs boson. The corresponding Feynman diagrams (at tree-level) are shown in Fig. 1.

It is interesting to note that in the case of complex parameters, γ and A^0 always contribute to the cross section. The total cross section (at tree-level) is given by

$$\sigma(\mu^+\mu^- \to \tilde{q}_i\bar{\tilde{q}}_j) = \frac{\pi\alpha^2 k_{ij}}{2s^2}\left\{\frac{2k_{ij}^2}{3s^2}T_{VV} + T_{HH} + \frac{m_i^2 - m_j^2}{2}T_{VH}\right\}, \tag{9}$$

where $\sqrt{s}$ is center-of-mass energy, $k_{ij} = \sqrt{(s - m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2)^2 - 4m_{\tilde{q}_i}^2 m_{\tilde{q}_j}^2}$. And

$$\begin{aligned} T_{VV} &= e_q^2\left|\delta_{ij}\right|^2(1 - P_-P_+) - \frac{e_q.\mathrm{Re}(c_{ij}d_Z\delta_{ij}^+)s}{2s_w^2c_w^2}\left[v_\mu(1 - P_-P_+) - a_\mu(P_- - P_+)\right] \\ &\quad + \frac{s^2|c_{ij}|^2|d_Z|^2}{16s_w^4c_w^4}\left[(v_\mu^2 + a_\mu^2)(1 - P_-P_+) - 2a_\mu v_\mu(P_- - P_+)\right], \end{aligned} \tag{10}$$

$$\begin{aligned} T_{HH} &= \frac{h_\mu^2 s}{2e^4}\Big\{\Big(\left|(G_1^\alpha)_{ij}\sin\alpha d_{h^0} - (G_2^\alpha)_{ij}\cos\alpha d_{H^0}\right|^2 + \left|(G_3^\alpha)_{ij}\sin\beta d_{A^0}\right|^2\Big)(1 - P_-P_+) \\ &\quad + (P_- + P_+)2\,\mathrm{Re}\Big[\Big((G_1^{\tilde{q}})_{ij}\sin\alpha d_{h^0} - (G_2^\alpha)_{ij}\cos\alpha d_{H^0}\Big)^+(G_3^\alpha)_{ij}\sin\beta d_{A^0}\Big]\Big\}, \end{aligned} \tag{11}$$

$$\begin{aligned} T_{VH} &= \frac{\sqrt{2}h_\mu m_\mu}{e^2}\Big\{2\,\mathrm{Re}\Big[\delta_{ij}\Big(\sin\beta(G_3^\alpha)_{ij}d_{A^0}\Big)^+\Big](P_- - P_+) \\ &\quad + \mathrm{Im}\Big[\Big((G_1^{\tilde{q}})_{ij}\sin\alpha d_{h0} - (G_2^\alpha)_{ij}\cos\alpha d_{H^0}\Big)^+ C_{ij}d_z\Big]\frac{(P_- + P_+)a_\mu}{2s_w^2c_w^2}\Big(1 - \frac{s}{M_Z^2}\Big) \\ &\quad - 2\,\mathrm{Re}\Big[\Big((G_3^\alpha)_{ij}\sin\beta d_{A^0}\Big)^+ C_{ij}d_z\Big]\Big(2a_\mu - v_\mu(P_- - P_+)\Big)\Big(1 - \frac{s}{M_Z^2}\Big)\Big\}, \end{aligned} \tag{12}$$

where $d_x = |(s - m_x^2) + i\Gamma_x m_x|^{-1}$ (with $x = Z, h^0, H^0, A^0$) and P_- is the polarization factor of the μ^- beam, P_+ that of the μ^+ beam.

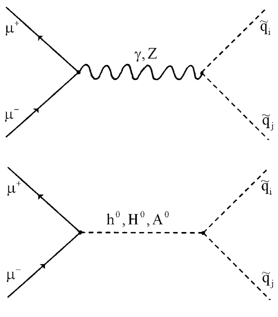

Fig. 1 Feynman diagrams for the process $\mu^+\mu^- \to \tilde{q}_i\bar{\tilde{q}}_j$

3 Numerical Results and Discussions

Let us now turn to the numerical analysis. Masses and couplings of Higgs boson depend on the parameters μ and $\tan\beta$. We take $m_{\tilde{t}_1} = 180$ GeV, $m_{\tilde{t}_2} = 256$ Gev, $\cos\theta_{\tilde{t}} = -0.55$, $m_{\tilde{b}_1} = 175$ GeV, $m_{\tilde{b}_2} = 195$ Gev, $\cos\theta_{\tilde{b}} = 0.9$, $\tan\beta = 3$, $M_{\tilde{E}} = 150$ GeV, $M_{\tilde{L}} = 170$ GeV, $m_{H^0} = 466$ GeV, $m_{A^0} = 450$ GeV, $\sin\alpha = -0.35$, $\Gamma_{H^0} = 5.4$ GeV, $\Gamma_{A^0} = 7.3$ GeV, $\sqrt{s} = 450$ GeV as input parameters. The sbottom masses and mixing angle are fixed by the assumptions $M_{\tilde{D}} = 1.12\ M_{\tilde{Q}}(\tilde{t})$, $|\mu| = 300$ GeV and $|A_t| = |A_b| = 300$ GeV.

We show in Figs. 2–7 the $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ dependence of the ratios σ_R/σ_C of unpolarized cross sections (with R and C indices corresponding to the case of real and complex parameters, respectively) of the processes: $\mu^+\mu^- \to \tilde{t}_i\bar{\tilde{t}}_j$, $\tilde{b}_i\bar{\tilde{b}}_j$ $(i, j = 1, 2)$. In

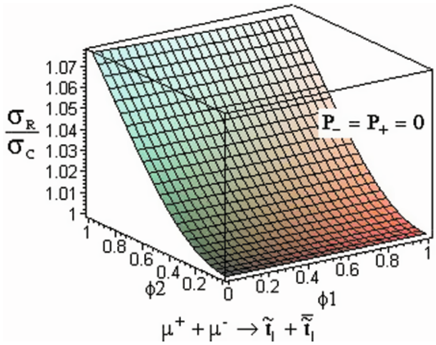

Fig. 2 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{t}_1\bar{\tilde{t}}_1$ for unpolarized μ^+, μ^- beams

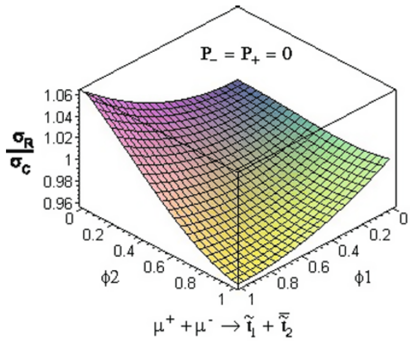

Fig. 3 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{t}_1\bar{\tilde{t}}_2$ for unpolarized μ^+, μ^- beams

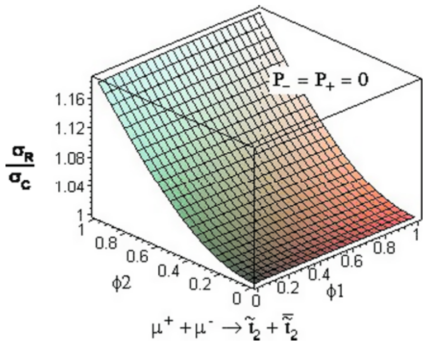

Fig. 4 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{t}_2\bar{\tilde{t}}_2$ for unpolarized μ^+, μ^- beams

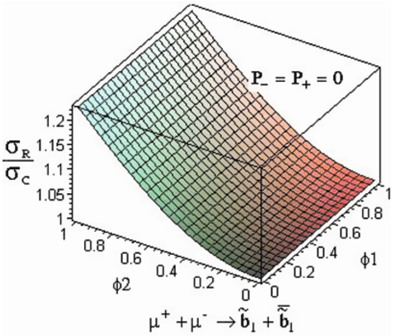

Fig. 5 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{b}_1\bar{\tilde{b}}_1$ for unpolarized μ^+, μ^- beams

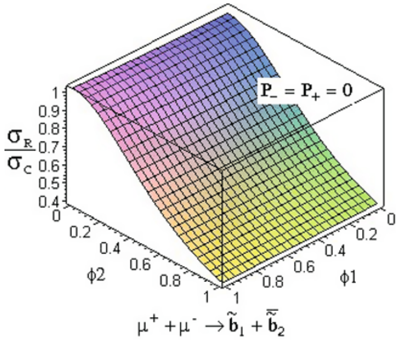

Fig. 6 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{b}_1\bar{\tilde{b}}_2$ for unpolarized μ^+, μ^- beams

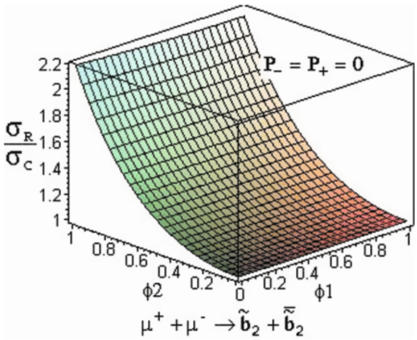

Fig. 7 Variation of ratio σ_R/σ_C with $\phi_1 = \phi_\mu$ and $\phi_2 = \phi_{A_q}$ of the process $\mu^+\mu^- \to \tilde{b}_2\bar{\tilde{b}}_2$ for unpolarized μ^+, μ^- beams

order to study the polarization effects on the cross section in case of complex parameters, we also plot in Figs. 8–13 the variation of the ratios σ_0/σ_P with polarization factors P_- and P_+ for specific values of ϕ_1, ϕ_2. Here the 0 and P indices correspond to the case of unpolarized and polarized beams respectively.

From Figs. 2–7 we can see that σ_R/σ_C exhibits explicit dependences on ϕ_2 while keeping nearly independent of ϕ_1 in most of processes $\mu^+\mu^- \to \tilde{t}_i\bar{\tilde{t}}_j, \tilde{b}_i\bar{\tilde{b}}_j$ except for $\mu^+\mu^- \to \tilde{t}_1\bar{\tilde{t}}_2$. In the range of ϕ_1 and ϕ_2 shown, the contribution of complex phases to the cross section $\Delta\sigma_C/\sigma_R = (\sigma_C - \sigma_R)/\sigma_R$ changes from -7% to 0% in case of $\tilde{t}_1\bar{\tilde{t}}_1$ production (Fig. 2); from -6% to 4% for $\tilde{t}_1\bar{\tilde{t}}_2$ production (Fig. 3); from 16% to 0% for $\tilde{t}_2\bar{\tilde{t}}_2$ production (Fig. 4); and is about from -18% to 0%, from 0% to 150% and from -54.4% to 0% for the productions of $\tilde{b}_1\bar{\tilde{b}}_1$ (Fig. 5), $\tilde{b}_1\bar{\tilde{b}}_2$ (Fig. 6), $\tilde{b}_2\bar{\tilde{b}}_2$ (Fig. 7), respectively.

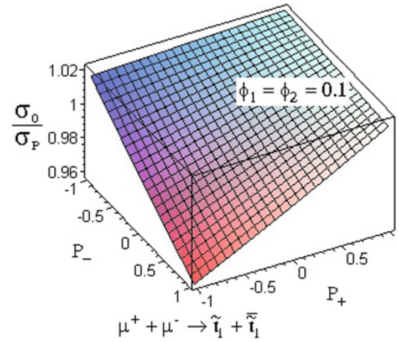

Fig. 8 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{t}_1\bar{\tilde{t}}_1$ for $\phi_1 = \phi_2 = 0.1$

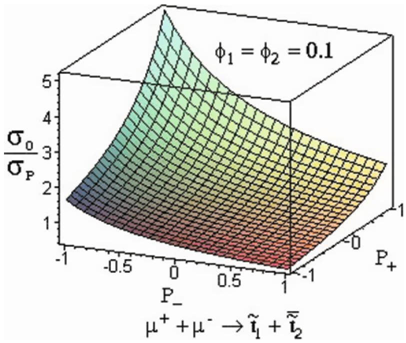

Fig. 9 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{t}_1\bar{\tilde{t}}_2$ for $\phi_1 = \phi_2 = 0.1$

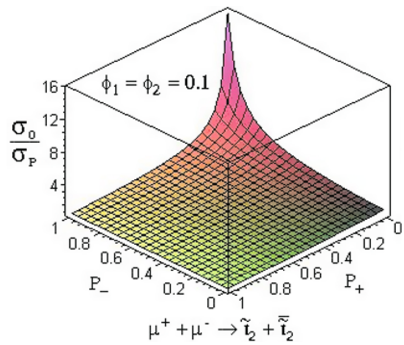

Fig. 10 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{t}_2\bar{\tilde{t}}_2$ for $\phi_1 = \phi_2 = 0.1$

The effect of polarizations P_-, P_+ on the cross section for specific values of $\phi_1 = \phi_2 = 0.1$ is strongest on the production of $\tilde{t}_2\bar{\tilde{t}}_2$ as dictated in Figs. 8–13 for $P_-, P_+ \in [-1, 1]$. It can suppress the cross section at most by 5 times in cases of $\tilde{t}_1\bar{\tilde{t}}_2$ or $\tilde{b}_1\bar{\tilde{b}}_1$ productions (Figs. 9 and 11); by about 16 times for $\tilde{t}_2\bar{\tilde{t}}_2$ production (Fig. 10) and by about 12 times for $\tilde{b}_1\bar{\tilde{b}}_2$ production (Fig. 12). In cases of $\tilde{t}_1\bar{\tilde{t}}_1$ and $\tilde{b}_2\bar{\tilde{b}}_2$ productions, P_-, P_+ can contribute to the unpolarized cross section from -2% to 4% for $\tilde{t}_1\bar{\tilde{t}}_1$ production (Fig. 8) and from -15% to 20% for $\tilde{b}_2\bar{\tilde{b}}_2$ production (Fig. 13).

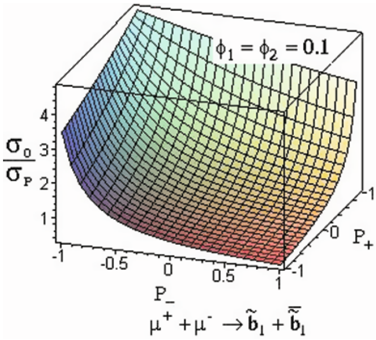

Fig. 11 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{b}_1\bar{\tilde{b}}_1$ for $\phi_1 = \phi_2 = 0.1$

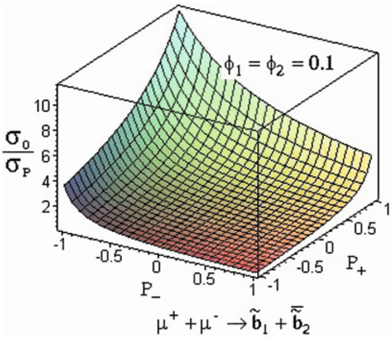

Fig. 12 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{b}_1\bar{\tilde{b}}_2$ for $\phi_1 = \phi_2 = 0.1$

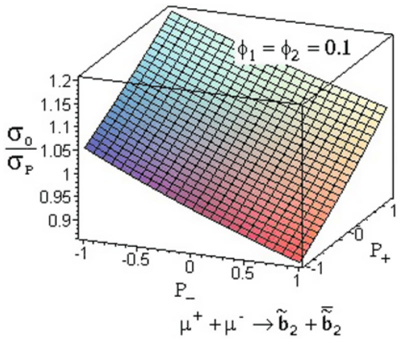

Fig. 13 Polarization dependence of the ratio σ_0/σ_P of the process $\mu^+\mu^- \to \tilde{b}_2\bar{\tilde{b}}_2$ for $\phi_1 = \phi_2 = 0.1$

4 Conclusions

In this paper, we have discussed the squark pair production in μ^+ μ^- collision within the MSSM with complex parameters μ, A_q. Tree-level results have been presented. The one-loop corrections to the cross section of these processes are left for a future work. We have also taken into account the polarization effects of the μ^+, μ^- beams. We have found that at tree-level the effects of the CP violating phases and of the beam polarizations can be quite strong. These could have important implications for the $\tilde{t}_i$ and $\tilde{b}_i$ searches and the MSSM parameter determination in future collider experiments. Works along these lines are in progress.

Acknowledgements We are grateful to Prof. G. Belanger for suggesting the problem and for her valuable comments. H.H. Bang wishes to thank Prof. P. Aurenche for his help and encouragement.

This work was supported in part by Project on Natural Sciences of Vietnam National University (The Strong Scientific Group on Theoretical Physics).

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